ELSEVIER

Decision support for the design of constructed wetlands

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One important component of computational mathematical modelling for industry is the actual construction of products that support the decision making of some specific industrial activity, such as the design of constructed wetlands to meet environmental or pollution control guidelines. In this paper attention is focused on the construction of user-friendly decision-support systems (products) and the inter-related matters of computational mathematical modelling. The specific industrial activity examined is the design of wetlands, lakes, and ponds to meet environmental and pollution guidelines. In particular we consider some of the computational mathematical modelling that underlies the development of the decision support for CSIRO's NESSIE. In fact, in the development of such a system, the driving force is, as explained in Anderssen, et al.,[1] the need to give the designer maximum flexibility to explore the various scenarios and options appropriate to the environmental and pollution guidelines under consideration. There are various ways in which this can be achieved. For the development of NESSIE, the goals were to give the designer access to a user-friendly computer system, which allows one to quickly and interactively determine and compare the horizontal dynamics of various lake configurations in terms of velocity, streamline, and residence time patterns; and freedom of choice in matching hydrodynamical indicators, such as velocity, streamlines, and residence times, with corresponding environmental and pollution models. The key to the implementation of the above proposal is the decoupling of the hydrodynamical modelling from the environmental and pollution modelling. Its clear advantage from a decision-support point of view is its recognition of the role and responsibility of the designer in the overall decision making associated with the planning and construction of wetlands. In this paper we examine the computational mathematical modelling rationale behind the proposal.

Keywords: decision support, constructed wetlands, depth-averaged flow

1. Introduction

When building a decision-support system, the key is to understand and clearly differentiate between the roles and responsibilities of the modeller and the decision maker. Thus, when developing decision support for the design of constructed wetlands to meet environmental and pollution control guidelines, it is necessary to draw a clear distinction between the hydrodynamical and environmental (and pollution) modelling. The nature of this separation is discussed in some detail in Section 2.

In a recent paper, Anderssen et al.[1] explained how this separation has been achieved with NESSIE; namely,

(1) Comparative assessment of the horizontal dynamics. Through the BUILD-SOLVE-DISPLAY-AMEND modularization of NESSIE, this allows the designer to quickly and interactively determine and compare the horizontal dynamics of various lake configurations in terms of velocity, streamline, and residence time patterns.
(2) Decoupling the environmental modelling from the hydrodynamical. This allows the designer freedom of

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Received 29 November 1993; accepted 24 July 1995

Appl. Math. Modelling 1996, Vol. 20, January

655 Avenue of the Americas, New York, NY 10010

0307-904X/96/$15.00
SSDI 0307-904X(95)00101-O

choice in matching hydrodynamical indicators, such as velocity, streamline, and residence times with corresponding environmental and pollution models (e.g., interpretation of partial residence times in the study of depositional patterns of sediments).

The advantages of this approach are that it allows the designer to focus on the environmental considerations relevant to the underlying decision-making and to match qualitative and quantitative modelling of the environmental considerations with the relevant hydrodynamical indicators (e.g., the use of streamline and velocity patterns in the determination of suitable locations for aquatic vegetation). This gives the designer maximum flexibility to explore the various scenarios and options appropriate to the wetlands design problem under consideration.

This approach does not exclude the possibility of using models where the hydrodynamics and the environmental considerations are strongly coupled in some nonlinear manner. However, the need for such sophistication in any given investigation can only be justified after suitable exploratory modelling and analysis has been performed. In addition, as is common in many practical situations, the aim is not an exact characterization of the process under consideration, which is likely to be time-consuming if not impossible, but some way of quickly assessing the overall picture which is all that is often required for decision-making purposes.

In developing NESSIE, the key considerations were its userability design, which lead naturally to the formulation of the BUILD-SOLVE-DISPLAY-AMEND modularization (a software engineering paradigm) around which NESSIE has been developed; and the underlying computational mathematical modelling, which aimed to guarantee decision support for a representative class of problems, namely linearized depth-averaged steady-state flow.

The former has been discussed in some detail in Anderssen.[2] It is the latter that is the focus of this paper. The aim is to describe in some detail the rationale behind the actual choice of computational mathematical modelling used in NESSIE. Initially, in Section 2, the importance of the decoupling of the hydrodynamics from the environmental (pollution) modelling is discussed. The computational mathematical modelling is examined in Section 3, where the depth-averaged modelling on which NESSIE is based is described. The paper ends with an assessment in Section 4 of possible choices for the matching of hydrodynamical indicators, such as velocity, streamline, and residence times with corresponding environmental and pollution models.

2. Modelling pollution processes in wetlands, ponds, and lakes

Put succinctly, the aim of modelling is to assist with answering questions. In this light the actual modelling used for a particular problem must yield a framework in which the relevant questions can be examined and answered efficiently and successfully. Among other strategies, model partitioning[3] plays an important role in determining such a framework. The key to its implementation is the identification of the link concepts which allow the relationship between the data and the required information to be broken into a series of appropriate submodels.

When investigating pollution processes in constructed wetlands, the modelling partitions naturally into two distinct though interrelated steps:

(1) The hydrology. This is the primary mechanism that drives the behavior of any water body. The final effect of any pollution process depends on how the geometry and physics of the lake determine its overall flow characteristics. Because the modelling reduces to solving the well-established properly posed equations for fluid (e.g., depth-averaged) flow, the assumptions are tight and rigorous, and the modelling control is strong.

(2) The pollution. Depending on the situation under examination, this aspect of the modelling changes from application to application. In addition the assumptions are more open to debate and so the modelling control is not very strong. As a direct result, the need arises for flexibility to explore the consequences of alternative models for the pollution process under consideration.

The underlying link concepts that connect (1) to (2) are the hydrodynamical properties of the flow such as the velocities, streamlines, and the residence times.

Independently, the underlying decision-support involves: (i) exploration (comparative assessment), and (ii) consolidation.

The purpose of NESSIE is initially to support (i) through (1), since the control is strong. Then, on the basis of decisions made at that stage about suitable lake configurations, support (i) and (ii) through (2). Clearly, such a strategy cannot be implemented if the indicators that link (1) to (2) for the problem under consideration have not been identified. In part this illustrates the crucial role played by the link concepts in model partitioning.

3. Modelling horizontal dynamics with NESSIE

NESSIE was developed to be an aid for decision-making connected with the design of artificial lakes for pollution control or related matters and issues. In the modelling and study of the dynamics of natural water bodies, the first step is to identify the dominant aspects of the dynamics in the application under consideration. For example, when examining the biological implications for marine life living at the bottom of a river or lake, the structure and behavior of the boundary layer becomes the key issue.[4] However, when mixing and stratification in a lake or river are the dominant characteristics, the vertical dynamics become the dominant issue.[5]

Here the emphasis is on situations where the dominant characteristics of the flow are its average horizontal dynamics. Such flows are important when examining pollution aspects of natural water bodies since, in many practical situations, it is the average horizontal characteristics that determines erosion, sedimentation, pollution removal by macrophytes, etc. Thus, the decision as to whether or not average horizontal dynamics are the appropriate model to use depends on the nature of the problem being examined in terms of the questions that need to be answered. Hence, if pollution characteristics of a natural water body are the essence of the decision making, then average horizontal dynamics are likely to play an important role in answering the relevant questions.

The essential strategy that motivates and underpins the development of NESSIE is the construction of a user-friendly computer system that interactively provides comparisons of the horizontal flow and residence time patterns for different lake configurations. This has been achieved through restricting attention to models for the flow which correspond to elliptic partial differential equations and through the use of state-of-the-art algorithms such as *PLTMG*[6] for their solution. Background to this strategy and to the decision to use comparative assessment as the framework in which to do the development of the user-friendly system can be found in Anderssen et al.[7] and Anderssen.[2]

The central idea of reducing the solution of flow equations to the solution of a related scalar elliptic partial differential equation is not new. It is the basis for Ekman's[8] paper on the influence of the Earth's rotation on ocean currents. It could be argued that it goes back to Laplace. Welander[9] uses the same idea, acknowledging Ekman[8] as the source. However, its potential in the design of codes for fluid flow problems has not been fully exploited.

There are various ways in which the depth-averaged flow equations can be reduced to the solution of elliptic partial differential equations. Welander[9] examines one strategy, while Wilders et al.[10] propose another. An alternative derivation for steady-state linearized depth-averaged flow is given in the next section. It is the basis for the NESSIE model.

3.1 Depth-averaged modelling of flow

The starting point for the present analysis are the equations that describe the linearized horizontal dynamics of natural water bodies. They were initially formulated by Ekman[8] using complex function notation. Mathematically they have a neat compact form which has proved quite useful in analyzing their properties and applying them to representative situations.[9,11] However, for computational purposes it is more appropriate to work with these equations in standard form, even though their presentation is a little cumbersome. The advantage of the standard form is that it makes explicit the essential structure for which algorithms must be developed. Here, the derivation and notation of Hunter and Hearn[11] is followed, since their paper gives a clear up-to-date picture of the use of depth-averaged (vertically averaged) equations in the modelling of the flow patterns in natural water bodies such as seas and lakes. Similar equations can be found in various sources including Proudman.[12,13]

The equations of linearized horizontal dynamics. For the linearized horizontal dynamics of natural water bodies, the basic equations are

$$\frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) + \frac{\partial \zeta}{\partial t} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} = fv + \frac{\partial}{\partial z}\left(N(z)\frac{\partial u}{\partial z}\right) - g\frac{\partial \zeta}{\partial x} \tag{2}$$

$$\frac{\partial v}{\partial t} = -fu + \frac{\partial}{\partial z}\left(N(z)\frac{\partial v}{\partial z}\right) - g\frac{\partial \zeta}{\partial y} \tag{3}$$

where (u, v) denote the components of the horizontal velocity $\mathbf{u}$ with coordinates x and y, respectively; $(\bar{u}, \bar{v})$ denote the depth-averaged components of the horizontal velocity $\mathbf{u}$; z denotes the vertical coordinate (such that $-h$ corresponds to the bottom of the lake, 0 to the mean depth, and ζ to the surface elevation); t denotes time; h denotes the depth at a point (x, y); ζ denotes the surface elevation at a point (x, y); f denotes the Coriolis parameter; and $N(z)$ denotes the vertical eddy viscosity.

In the above formulation, the continuity equation (namely equation [1]) is already vertically averaged to ensure that the vertical velocity component w is removed from explicit consideration. In addition, variations in the atmospheric pressure, which are included in the Proudman[12] formulation, have been ignored. The nonlinear terms in the general equations of motion have been neglected.[12,13] For example, such circumstances hold when the Rossby number R_0 for the flow is small[14] (Section 1.2), which corresponds for example to the Coriolis forces dominating the nonlinear accelerations.

Integration of equations (2) and (3) with respect to z and division by h yields

$$\frac{\partial \bar{u}}{\partial t} = f\bar{v} + \frac{1}{\rho h}(S^{(u)} - B^{(u)}) - g\frac{\partial \zeta}{\partial x} \tag{4}$$

$$\frac{\partial \bar{v}}{\partial t} = -f\bar{u} + \frac{1}{\rho h}(S^{(v)} - B^{(v)}) - g\frac{\partial \zeta}{\partial y} \tag{5}$$

where

$$\frac{S^{(u)}}{\rho} = N(0)\left[\frac{\partial u}{\partial z}\right]_{z=0} \qquad \frac{B^{(u)}}{\rho} = N(-h)\left[\frac{\partial u}{\partial z}\right]_{z=-h} \tag{6}$$

$$\frac{S^{(v)}}{\rho} = N(0)\left[\frac{\partial v}{\partial z}\right]_{z=0} \qquad \frac{B^{(v)}}{\rho} = N(-h)\left[\frac{\partial v}{\partial z}\right]_{z=-h} \tag{7}$$

with $S^{(u)}$ and $B^{(u)}$ ($S^{(v)}$ and $B^{(v)}$) denoting the surface and bottom stresses associated with the velocity component u (v). The counterpart of equations (4) and (5) is equation (2.3) in Proudman.[12]

The surface boundary condition has been linearized to apply at $z = 0$ instead of at $z = \zeta$, since ζ is usually quite small relative to the size of h.

The linearity and physics of equations (4) and (5) can now be exploited. Because the surface stresses (due to the wind) and the surface slopes (due to inflow and outflow balance, as well as atmospheric pressure differences when they apply) can be assumed to be independent physical processes, the associated bottom stresses $B_1^{(u)}$, $B_1^{(v)}$ and $B_2^{(u)}$, $B_2^{(v)}$ must be independent and satisfy

$$B^{(u)} = B_1^{(u)} + B_2^{(u)} \qquad B^{(v)} = B_1^{(v)} + B_2^{(v)}$$

Thus, the velocity $\bar{\mathbf{u}}$ can be decomposed into components $\bar{u}_1$ and $\bar{v}_1$ driven by the surface stresses $S^{(u)}$ and $S^{(u)}$ and components $\bar{u}_2$ and $\bar{v}_2$ driven by the surface slopes $\partial\zeta/\partial x$ and $\partial\zeta/\partial y$. Formally, this allows equations (4) and (5) to be rewritten as:

$$\frac{\partial \bar{u}_1}{\partial t} = f\bar{v}_1 + \frac{1}{\rho h}\left(S^{(u)} - B_1^{(u)}\right) \tag{8}$$

$$\frac{\partial \bar{v}_1}{\partial t} = -f\bar{u}_1 + \frac{1}{\rho h}\left(S^{(v)} - B_1^{(v)}\right) \tag{9}$$

and

$$\frac{\partial \bar{u}_2}{\partial t} = f\bar{v}_2 - \frac{1}{\rho h}\left(B_2^{(u)}\right) - g\frac{\partial \zeta}{\partial x} \tag{10}$$

$$\frac{\partial \bar{v}_2}{\partial t} = -f\bar{u}_2 - \frac{1}{\rho h}\left(B_2^{(v)}\right) - g\frac{\partial \zeta}{\partial y} \tag{11}$$

where

$$\bar{\mathbf{u}} = \bar{\mathbf{u}}_1 + \bar{\mathbf{u}}_2 \tag{12}$$

As shown by Jelesnianski,[15] Appendix A, it follows from a standard Laplace transform analysis of equation (2) and (3) that the surface stresses and the surface slopes are related to the bottom friction by the convolution integral equations

$$B_1^{(u)} = \int_0^t C_1^{(u)}(t) S^{(u)}(t-\tau)\, \mathrm{d}\tau \tag{13}$$

$$B_1^{(v)} = \int_0^t C_1^{(v)}(t) S^{(v)}(t-\tau)\, \mathrm{d}\tau \tag{14}$$

$$B_2^{(u)} = \int_0^t C_2^{(u)}(t) \frac{\partial \zeta}{\partial x}(t-\tau)\, \mathrm{d}\tau \tag{15}$$

$$B_2^{(v)} = \int_0^t C_2^{(v)}(t) \frac{\partial \zeta}{\partial y}(t-\tau)\, \mathrm{d}\tau \tag{16}$$

where the kernels $C_j^{(u)}$, $C_j^{(v)}$, $j = 1, 2$, are derived form the equations (4–7) with a suitable choice for the initial conditions.

This linear partitioning of the equations (4) and (5) dates back to Welander[9] and has been exploited in various ways by a number of authors including Forristall,[16] Jelesnianski,[15] Jordan and Baker,[17] and Hearn and Hunter.[18] However, as is clear from the structure of equations (7)–(11), the two surface stress equations, as well as the two surface slope equations, remain coupled through the Coriolis parameter f.

Conventional bottom friction. As explained in Hunter and Hearn[11] (p. 203), if the conventional bottom friction relationship is used, then $B_j^{(u)}$ and $B_j^{(v)}$, j = 1, 2, take the form

$$B_j^{(u)} = \rho V \bar{u}_j \qquad B_j^{(v)} = \rho V \bar{v}_j \qquad j = 1, 2 \tag{17}$$

where V denotes the linear friction factor. The corresponding counterparts to equations (8)–(11) become

$$\frac{\partial \bar{u}_1}{\partial t} = f\bar{v}_1 + \frac{1}{\rho h}\left(S^{(u)} - \rho V \bar{u}_1\right) \tag{18}$$

$$\frac{\partial \bar{v}_1}{\partial t} = -f\bar{u}_1 + \frac{1}{\rho h}\left(S^{(v)} - \rho V \bar{v}_1\right) \tag{19}$$

and

$$\frac{\partial \bar{u}_2}{\partial t} = f\bar{v}_2 - \frac{V}{h}\bar{u}_2 - g\frac{\partial \zeta}{\partial x} \tag{20}$$

$$\frac{\partial \bar{v}_2}{\partial t} = -f\bar{u}_2 - \frac{V}{h}\bar{v}_2 - g\frac{\partial \zeta}{\partial y} \tag{21}$$

In terms of the above notation, Proudman,[12] equation (2.5), assumes that the bottom stresses satisfy

$$B_j^{(u)} = 2\rho k h \bar{u}_j \qquad B_j^{(v)} = 2\rho k h \bar{v}_j \qquad j = 1, 2 \tag{22}$$

where k denotes the coefficient of friction. These relationships are clearly different from those for conventional bottom friction, namely equation (17). Thus, in situations where h is more or less constant and $2kh \sim V$, these two forms of friction will yield similar results. However, when h varies greatly, the flow patterns corresponding to equations (17) and (22), respectively, will differ considerably, even if $2k\hat{h} \sim V$ where $\hat{h}$ denotes the average value of h over the lake. In fact, the flows determined by equations (22) will have faster velocities in the shallow regions of the lake and slower velocities in the deeper regions than the flows determined by equation (17).

The steady state. The steady-state counterparts of equations (18)–(21) are

$$\frac{V}{h}\bar{u}_1 - f\bar{v}_1 = \frac{1}{\rho h} S^{(u)} \tag{23}$$

$$\frac{V}{h}\bar{v}_1 + f\bar{u}_1 = \frac{1}{\rho h} S^{(v)} \tag{24}$$

and

$$\frac{V}{h}\bar{u}_2 - f\bar{v}_2 = -g\frac{\partial \zeta}{\partial x}, \tag{25}$$

$$\frac{V}{h}\bar{v}_2 + f\bar{u}_2 = -g\frac{\partial \zeta}{\partial y}. \tag{26}$$

Neglecting the Coriolis terms. If it is also assumed that the lake is not large, and therefore that the Coriolis terms (as well as the nonlinear terms neglected previously) are small compared with the others so that they can be neglected in equations (23)–(26), then these equations decouple to yield

$$\bar{u}_1 = \frac{1}{\rho V} S^{(u)} \tag{27}$$

$$\bar{v}_1 = \frac{1}{\rho V} S^{(v)} \tag{28}$$

and

$$\bar{u}_2 = -\frac{hg}{V}\frac{\partial \zeta}{\partial x} \tag{29}$$

$$\bar{v}_2 = -\frac{hg}{V}\frac{\partial \zeta}{\partial y} \tag{30}$$

Substitution of these results into the steady-state version of the continuity equation (1) then yields

$$\frac{\partial}{\partial x}\left(\frac{gh^2}{V}\frac{\partial \zeta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{gh^2}{V}\frac{\partial \zeta}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{h}{\rho V} S^{(u)}\right) + \frac{\partial}{\partial y}\left(\frac{h}{\rho V} S^{(v)}\right) \tag{31}$$

Thus, the problem is reduced to solving a standard elliptic partial differential equation where the wind effect enters as the nonhomogeneous term. It shows clearly how the terms associated with the surface elevation and the wind enter the equation. In particular, it indicates that when wind effects are negligible the flow patterns are determined by a homogeneous linear elliptic equation of the form of (2.6) in Proudman[12], where the coefficients take account of the nature of the bottom friction.

Steady-state Coriolis equations. Because the steady-state equations (23)–(26) are algebraic, they can be solved for u_1, v_1, u_2, and v_2 even when the Coriolis parameter f is not negligible. In fact, one obtains

$$\bar{u}_1 = \frac{1}{\rho h \gamma}\left(\frac{V}{h} S^{(u)} + f S^{(v)}\right) \tag{32}$$

$$\bar{v}_1 = \frac{1}{\rho h \gamma}\left(\frac{V}{h} S^{(v)} - f S^{(u)}\right) \tag{33}$$

$$\bar{u}_2 = -\frac{g}{\gamma}\left(\frac{V}{h}\frac{\partial \zeta}{\partial x} + f\frac{\partial \zeta}{\partial y}\right) \tag{34}$$

$$\bar{v}_2 = -\frac{g}{\gamma}\left(\frac{V}{h}\frac{\partial \zeta}{\partial y} - f\frac{\partial \zeta}{\partial x}\right) \tag{35}$$

where

$$\gamma = \frac{V^2}{h^2} + f^2$$

When equations (32)–(35) are substituted into the steady-state version of the continuity equation (1), one obtains the following elliptic partial differential equation for the flow

$$\frac{\partial}{\partial x}\left(\frac{gV}{\gamma}\frac{\partial \zeta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{gV}{\gamma}\frac{\partial \zeta}{\partial y}\right) + \frac{\partial}{\partial x}\left(\frac{ghf}{\gamma}\right)\frac{\partial \zeta}{\partial y} - \frac{\partial}{\partial y}\left(\frac{ghf}{\gamma}\right)\frac{\partial \zeta}{\partial x} = F(x, y) \tag{36}$$

where

$$F = \frac{\partial}{\partial x}\left(\frac{h}{\rho\gamma}\left[\frac{V}{h} S^{(u)} + f S^{(v)}\right]\right) + \frac{\partial}{\partial y}\left(\frac{h}{\rho\gamma}\left[\frac{V}{h} S^{(u)} - f S^{(v)}\right]\right) \tag{37}$$

Equations (36) and (37) yield the explicit counterpart of equation (37) in Hunter and Hearn.[11] This result appears to be new, in that Hunter and Hearn[11] (partly because they worked with the compact complex notation) and others failed to appreciate that the steady-state equations corresponding to (18)–(21) could be solved analytically for the velocity components. In Hunter and Hearn,[11] the terms entering their equation (37) are only known implicitly. It is interesing to note that though the essential idea underlying the NESSIE strategy (outlined in Section 1) goes back to Ekman[8] and has been utilized by some authors such as Welander,[9] its full potential as outlined above has not been exploited.

The above result confirms that the solution of a wide range of depth-averaged flow problems can be reduced to the solution of linear elliptic partial differential equations to which packages, like PLTMG, are directly applicable.

In addition, this approach has a clear advantage over the convolution approach of Hunter and Hearn[11] in that, if one is willing to specify in advance the form of the bottom friction (such as the conventional one examined above), it is only necessary to solve some associated elliptic partial differential equation and thereby circumvent the need to apply their convolution method. For a lake of constant depth for which it can be assumed that f and V are constant, the elliptic operator simplifies to a more standard form.

3.2 The NESSIE model

In the current version of NESSIE, the following steady-state model for the flow is used

$$\operatorname{div}(h\beta \operatorname{grad} \zeta) = F \text{ in } \Omega \text{(the lake [not including islands])} \tag{38}$$

$$-h\beta\frac{\partial \zeta}{\partial \mathbf{n}} = f(s) \text{ on } \partial\Omega_0 \text{(the outer lake boundary [excluding islands])} \tag{39}$$

$$\frac{\partial \zeta}{\partial \mathbf{n}} = 0 \text{ on } \partial\Omega_i \text{ (the island boundaries } i = 1, 2, \ldots, N) \tag{40}$$

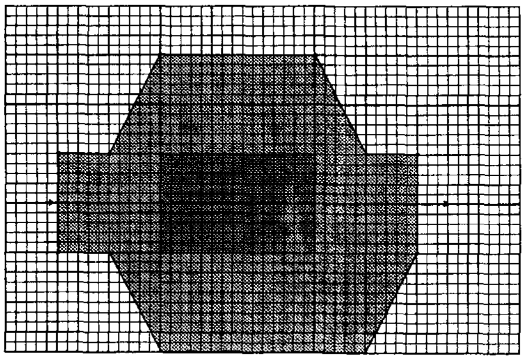

Figure 1. Constant bottom lake with resistive reed bed in the center.

where ζ denotes the surface elevation of the flow, h the depth of the lake, β the two-dimensional resistance to the flow, s arc-length, $f(s)$ the specified flows at the inlets and outlets, F a specified forcing function, and $\mathbf{n}$ the unit outward normal on the boundaries of the lake and islands. Clearly, the functions h and β will be strictly positive on Ω. For the linear frictional model of resistance[11] (equation [33]), β takes the form gh/V, where g and V denote, respectively, the acceleration due to gravity (9.807 m/s) and the linear friction factor (0.01–0.05 m/s for most small to medium sized lakes). Since it follows from equation (1) of the Appenix that the forcing term F (m/s) corresponds to the changing surface elevation as a function of time, it is the place where the evaporation, seepage, and wind effects are incorporated into the modelling. The form of F for a constant wind stress is given in Section 3.

The associated solvability condition, which guarantees consistency with the underlying steady-state assumption, is given by

$$\int_{\partial\Omega_0} f(s)\,\mathrm{d}s = \int_{\Omega} F(x)\,\mathrm{d}x \tag{41}$$

For the above model, the local fluid particle velocity $\mathbf{u}_p$ is given by

$$\mathbf{u}_p = -\beta\nabla\zeta + \boldsymbol{\tau}/\rho V \tag{42}$$

where the vector $\boldsymbol{\tau}$ models the surface stress generated by the wind. In the actual computer implementation of the above model, the stream function formulation corresponding to equations (1)–(5) is solved.[19] This guarantees that the resulting streamline patterns are sufficiently accurate to look hydrologically realistic. This is the major advantage of depth-averaged modelling over one dimension in that the latter must assume where the unknown streamlines are positioned.[20]

3.3 The advantages of NESSIE

From the point of view of the current discussion, the major advantages of NESSIE are:

- The ease with which it facilitates comparative assessment through its BUILD-SOLVE-DISPLAY-AMEND modularization. The decision-maker BUILDs an initial lake design, SOLVEs and DISPLAYs its hydrodynamics, and then AMENDs the original design to repeat the process and compare the consequences of the changes implemented through the use of AMEND. On a SPARC 2, this more or less happens in real time. The only slight delay is the 5–10 min for SOLVE to produce its results.
- The highly accurate streamline patterns it generates through the use of the stream-function formulation for the underlying hydrodynamical problem. For various reasons, the competitors of NESSIE cannot generate such high quality streamline patterns. Hydrodynamically this is their greatest limitation. In part this must be the reason they do not explicitly draw the clear distinction, made in the Introduction, between the hydrodynamics and the matching with the environmental decision making.

4. Matching the hydrodynamics with corresponding environmental aspects

As explained previously, from an environmental point of view, the key component in the design of constructed wetlands is the matching of the hydrodynamics with the relevant aspects underlying the design. It is not the hydrodynamical modelling itself that is an issue for the designer but the effect the hydrodynamics of the water body has on the environmental and pollution processes under consideration. For example, the shape of the plume from a point source varies greatly with the nature of the flow in the vicinity of the source.

The following examples of the matching process are discussed in Anderssen et al[1]: partial residence times and

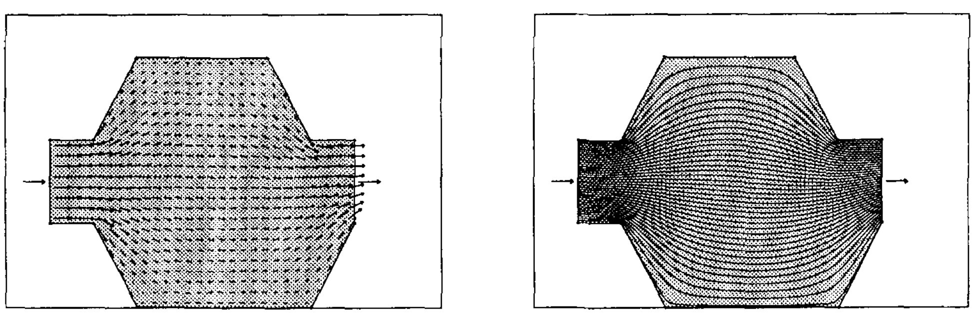

Figure 2. Velocity and streamline patterns when resistance of reed bed is the same as that at the bottom of the lake.

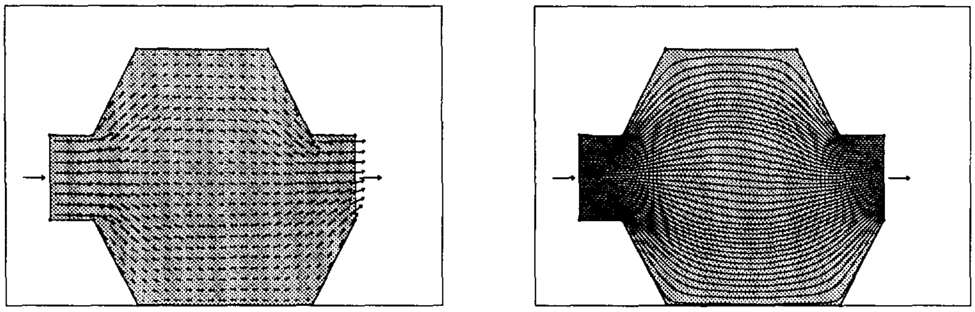

Figure 3. Velocity and streamline patterns when resistance of reed bed is twice that at the bottom of the lake.

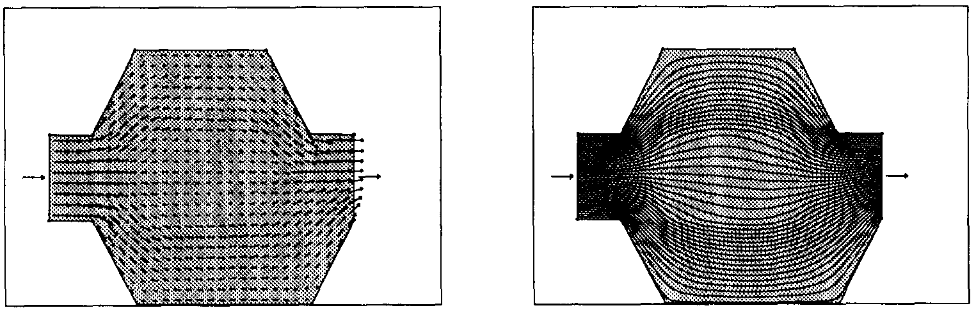

Figure 4. Velocity and streamline patterns when resistance of reed bed is three times that at the bottom of the lake.

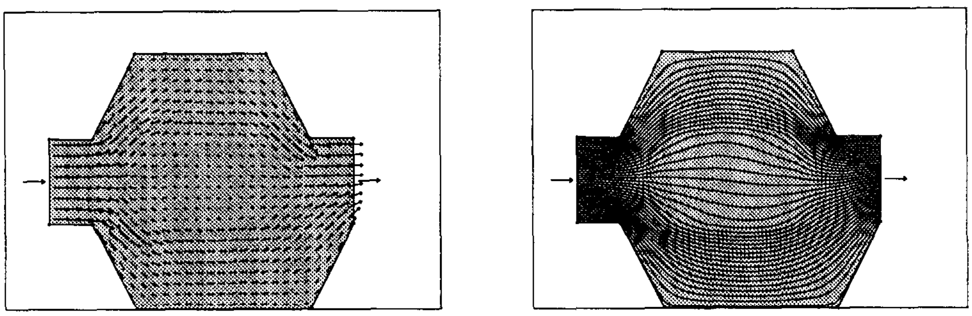

Figure 5. Velocity and streamline patterns when resistance of reed bed is four times that at the bottom of the lake.

sediment depositional patterns, streamlines and the location of aquatic vegetation, overall velocity and streamline patterns and flushing characteristics, and local velocity pattern and erosion.

Such examples clearly illustrate the basic steps in environmental decision making connected with the design of wetlands, ponds, and lakes to achieve predetermined goals. After specifying the goals, one first identifies the corresponding environmental features and the questions that must be answered about them in order to achieve the goals. Next, one relates them to the hydrodynamical indicators that contain the relevant information required for the decision making. Finally, one compares the values of these indicators for various lake configurations in order to find lake designs for which the goals are achieved. Formally, one is using a trial-and-error method to solve the underlying lake design problem (a parameter identification [inverse] problem).

Here, we examine the effect of resistance in the constructed wetland due to the presence of shallow zones and the presence of various types of vegetation. The figures illustrate that the effect of the resistance due to different vegetation types in a wetland tend to be small and therefore can be more easily assessed using streamline patterns than velocity patterns. *Figure 1* shows a constant bottom lake with a resistive reed bed in the center. *Figure 2* shows the velocity and streamline patterns when the problem is solved with the resistive pattern of the reed bed set to be the same as that at the bottom of the lake (i.e., no resistance). This solution corresponds, as one would expect, to the situation where one solves for the velocity and streamline patterns in the lake with no resistive term present. *Figures 3*, *4* and *5* show the velocity and streamline patterns for the situations where the resistive zone is twice, three times, and four times larger than that of the surrounding lake. They clearly illustrate the advantage of the streamline pattern over the velocity in interpreting the effects of the resistance zone at the center of the lake.

It is true that some environmental and pollution processes must be modelled by strongly coupled nonlinear equations when highly accurate assessments are required. Nevertheless, the approach described above plays an important exploratory role by allowing one to develop an intuitive understanding of the important factors in a particular situation as well as to compare alternatives for the modelling to be matched (coupled) with the hydrodynamics. In addition, the uncertainty about the geometry and physics of a lake call into question the use of unnecessary sophistication (see the last paragraph of the Introduction). The risk of too much sophistication is that it may produce artifacts of the modelling as if they were reality. The need in modelling to balance sophistication against achieving valid insight is always there no matter what the situation.

As the above discussion makes clear, environmental decision making must focus on the matching of the hydrodynamics with the particular issues under examination. In this way, the crutch is the hydrodynamical indicators such as velocities, streamlines, and residence times, not the modelling used to determine them as long it is accurate and reliable and performed on a user-friendly platform that allows the decision maker to focus on the issues under consideration. This has been the motivation and rationale for the development of NESSIE.

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